Design Optimisation of Locally Resonant Metamaterials under Uncertainty

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What is a Metamaterial?

Any structure that is architected to exhibit properties that go beyond the ones found in nature is defined as a Metamaterial. They often consist of periodic bodies with a repeating unit cell.

They are categorised based on the design goal as [1]:

- Acoustic Metamaterials: designed to manipulate sound waves in different ways, including absorption or deviation (Figure 1).
- Electro-magnetic Metamaterials: designed to manipulate light and electromagnetic waves, including negative refraction e.g., invisibility cloak.
- Mechanical Metamaterials: designed to perform in unusual ways under tension or compression, e.g., negative Poisson's ratio.

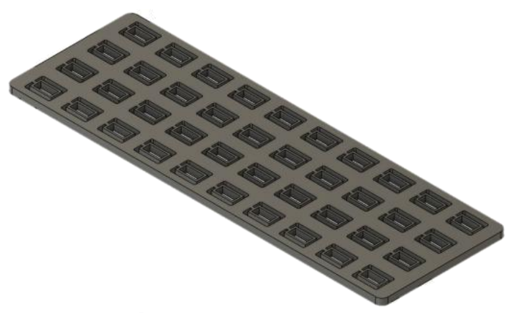
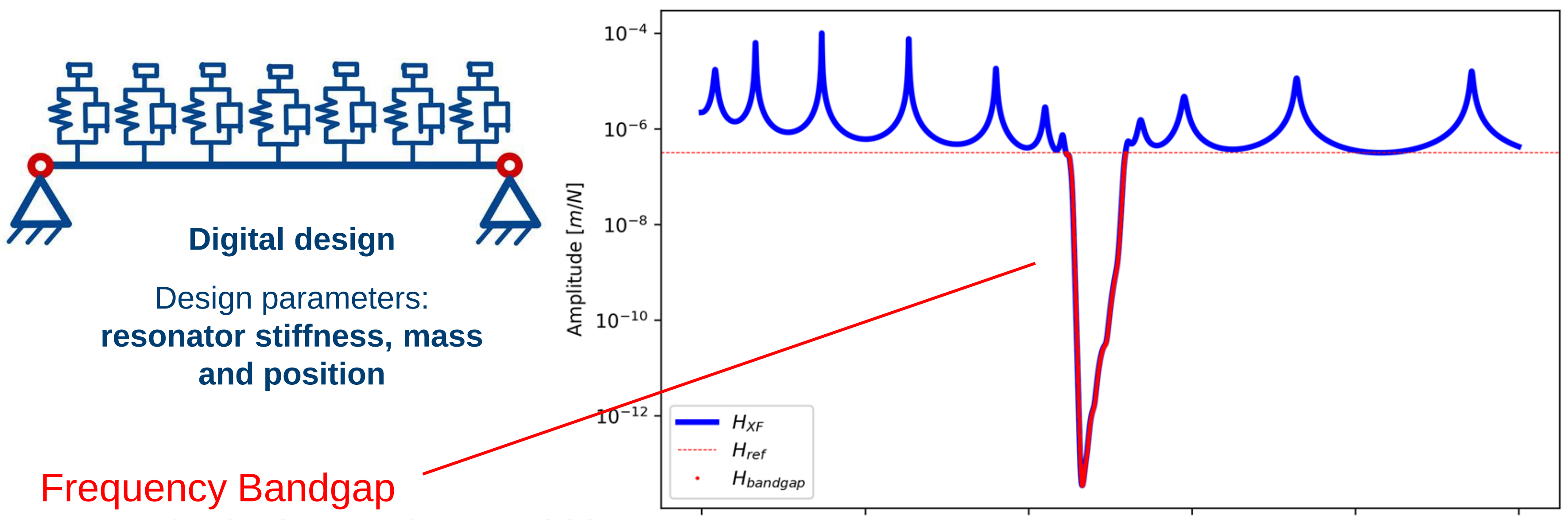


Figure 1

Locally Resonant Acoustic Metamaterials

LRAMs are a particular type of Acoustic Metamaterial that aims at absorbing noise and vibrations at targeted frequencies by exploiting local resonances of an array of specifically tuned unit cell. On a Frequency Response Function, this translates into a drop in the amplitude, known as frequency bandgap (as shown in Figure 2).



Digital design

Design parameters:
resonator stiffness, mass and position

Frequency Bandgap
Goal: obtain the maximum width

Figure 2

Challenge: Uncertainties in LRAMs

Discrepancies between the digital design and the real-world system (even if **nominally identical**) arise from various sources, including:

- Manufacturing:
  - Variability in geometrical dimensions of unit cells
  - Variability between different manufacturing methods
  - Mounting
- Modelling:
  - Periodicity of unit cell assumption
  - Single DOF mass resonator model
- Integration of measurements and models:
  - Non-deterministic inputs
  - Experimental noise

This translates into an **interval for the design parameters** of the LRAM beam and an interval in the frequency bandgap width.

Real-world system

Analysis of the interval of **frequency bandgap width** for intervals of design parameters

Figure 3

Finding an optimal design

Different **optimisation techniques** can be used to find a design that maximises the frequency bandgap:

- Global search algorithms:
  - They require a very large number of model runs.
  - Not feasible for expensive LRAM physics-based models.
- Machine Learning approach – **Bayesian optimisation** [2]:
  - The maximum bandgap width is searched by fitting a Gaussian Process to the function relating it to the resonator design parameters.
  - The GP is gradually updated by selecting a point that is believed to be the closest to the global maximum.
  - Significantly reduces the number of model runs required.

Challenge: finding the True Optimal

The main scope of the project is to realise an optimisation framework able to accept inputs from both deterministic model runs and results from experiments of the same nominal configuration. This novel framework will allow to:

- Account for uncertainties in the optimisation by including experimental data.
- Reduce prototype testing by running the optimisation loop on a limited number of simulations, test experimentally a sampled sub-optimal design and re-running the optimisation loop including the newly obtained experimental data.

Results of optimisation on interval analysis

Bayesian Optimisations on the system shown in Figure 1, which acts as a simple model of a LRAM, was performed. Results are shown from a FEM with Euler Bernoulli beam elements and 10 tuned mass dampers. Multiple configurations for different unit cells nominal resonant frequencies (Figure 4) and different nominal damper masses (Figure 5) were tested to find the Lower and Upper bounds of the bandgap width. Variability of the resonator stiffness is analysed, with a variability of ± 5% around the nominal value.

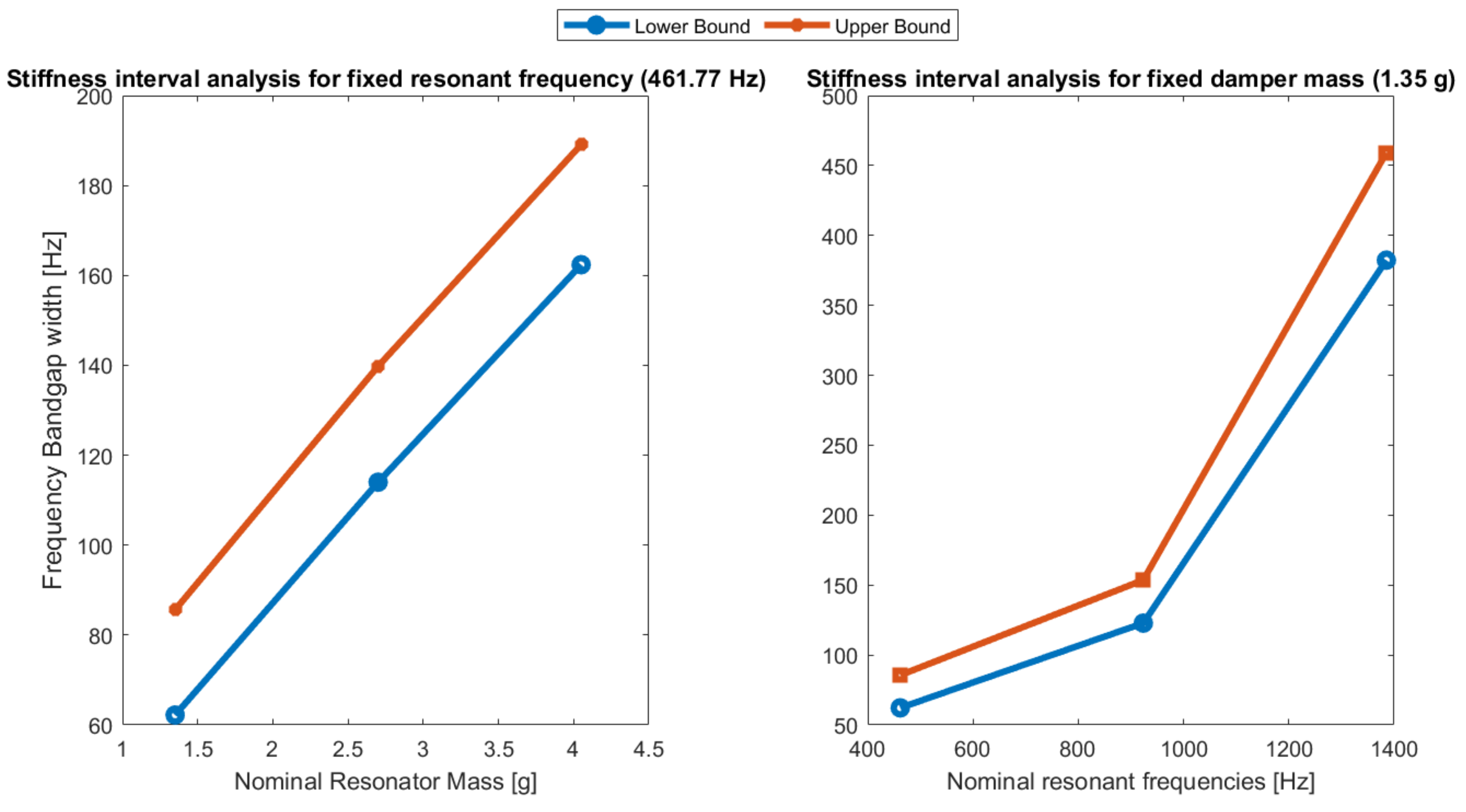


Figure 4

Figure 5

Future work

Short term: Collection of experimental data from different beams with arrays of resonators that are nominally identical in their design parameters but are realised differently by using different resonator shapes, manufacturing processes and mounting techniques.

Long term: realise an optimisation framework able to process inputs from both deterministic models and experimental tests.

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References

[1] G. Aydın and S. E. San, "Breaking the limits of acoustic science: A review of acoustic metamaterials," Materials Science and Engineering B, vol. 305, pp. 117384–117384, Jul. 2024.

[2] L. V. Belle, E. Deckers, and A. Cicirello, "Investigating and exploiting the impact of variability in resonator parameters on the vibration attenuation in locally resonant metamaterials," Philosophical Transactions of the Royal Society A Mathematical Physical and Engineering Sciences, vol. 382, no. 2279, Aug. 2024.