Design Optimisation of Locally Resonant Metamaterials under Uncertainty

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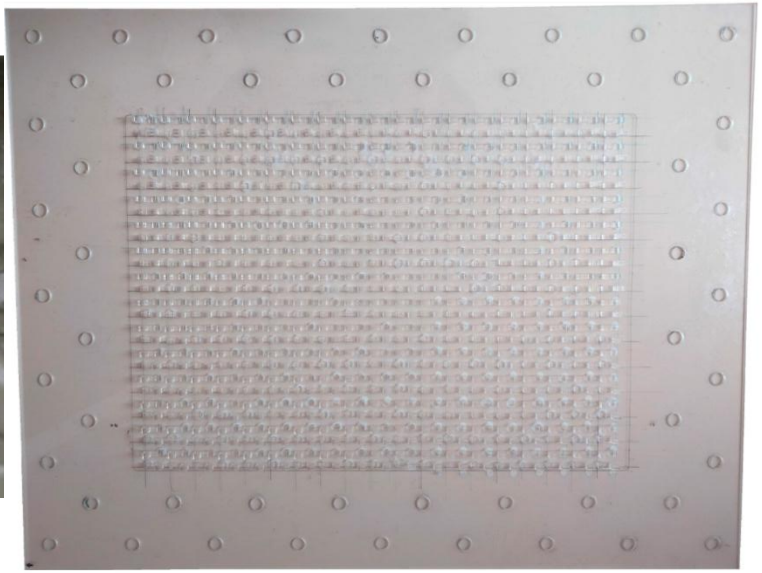
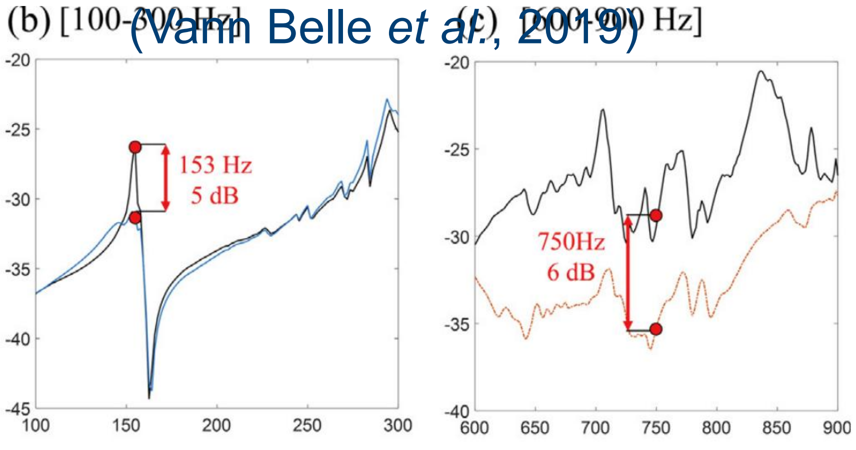
Locally Resonant Metamaterials (LRMs)

- A Metamaterial is a material that is engineered and built to exhibit properties that cannot be found in nature (Aydin *et al.*, 2024).
  - **Frequency bandgap at a resonant frequency**
- Metamaterials for vibrations absorption:
  - Can have **local resonators tuned to create a bandgap in the frequency response function** at targeted frequencies
- They usually consist of periodic arrays of unit cells
- LRMs could be:
  - 3D printed structures with embedded resonators
  - Resonators addition to a pre-existing structure



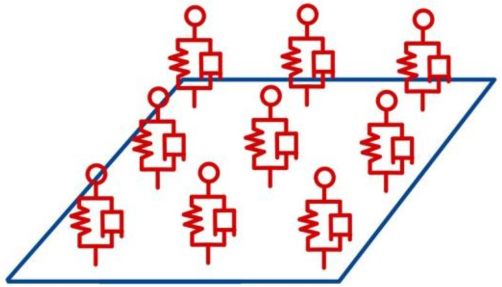
(Van Belle *et al.*, 2019)



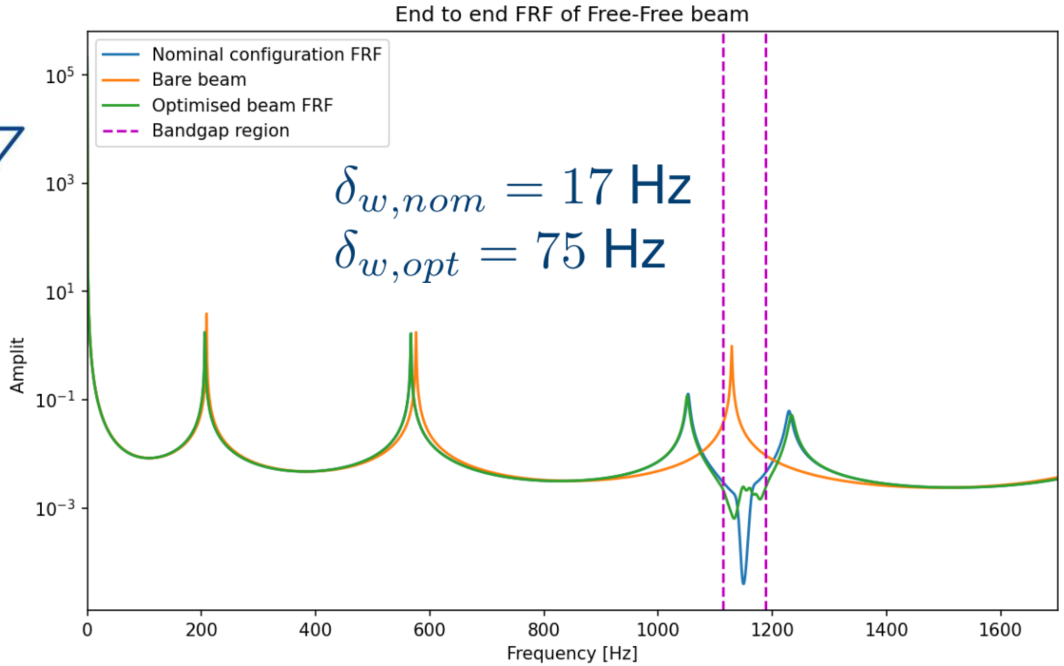
Jung *et al.*, 2018

How to design optimal LRMs?

- Optimisation of the local resonators to obtain the largest bandgap and absorb the largest spectrum of frequencies possible



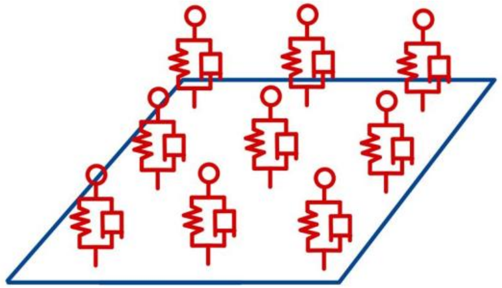
- Input parameters:
  - k_{res}, stiffness of resonator(s)
  - m_{res}, mass of resonator(s)
  - x_{res}, position of resonator(s) on the original system



- Output parameter:
  - δ_w, width of frequency bandgap
  - Defined as the region of the FRF below the FRF of the same structure without resonators

Bottlenecks in design optimisation of LRMs due to uncertainties

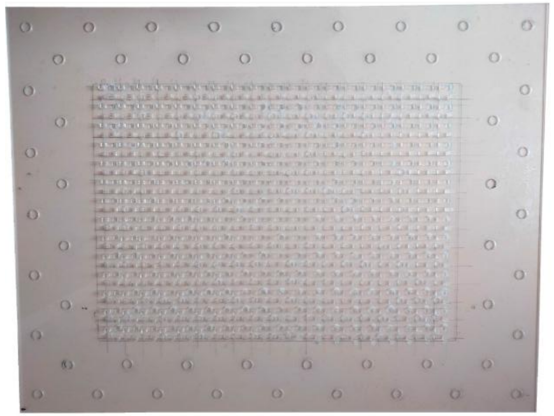
Optimisation approaches work well for models but do not keep uncertainties of real world systems into account!



Digital design

Parameters: resonator stiffness, mass and location

Optimisation against **frequency bandgap width**



Real world system
(Vann Belle *et al.*, 2019)

Assumptions in modelling

- Unit cell modelling
- **Approximation of a LRM resonator as a SDOF**

Manufacturing

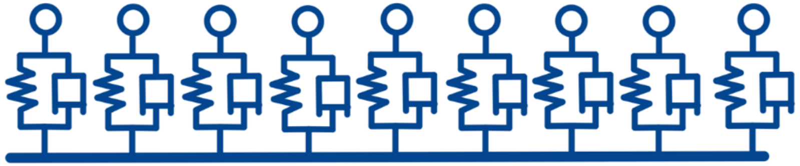
- **Geometry variability**
- Variability between different manufacturing methods
- **Boundary condition**

Experimental setup

- **Measurements quality**
- **LRM Boundary condition**
- **Location and direction of driving force**

Case study for characterisation of uncertainty of LRMs

- Structural beam with an unwanted resonant frequency at 1200 Hz
  - Mildd steel, E = 200 GPa, ρ = 8323 kg/m^3, 315 x 20 x 3 mm^3

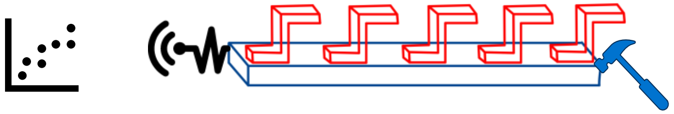


- Goal: design optimisation of a simple structure accounting for uncertainty:
  - **A beam with resonators placed on top** to create a bandgap around the resonant frequency
- How the problem is tackled:

1. Build prototype of a starting Configurations and test it with Experimental Modal Analysis for model validation

2. Design optimisation of a beam with resonators via a Finite Element Model and Bayesian Optimisation (Van Belle *et al.*, 2024)

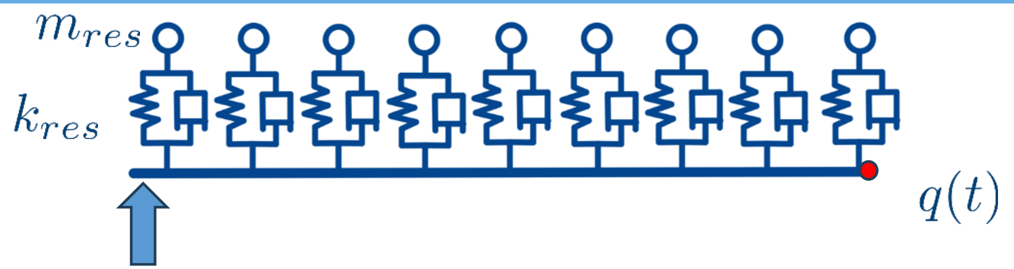
3. Build prototypes of the Nominal Optimal Configurations and test it with Experimental Modal Analysis



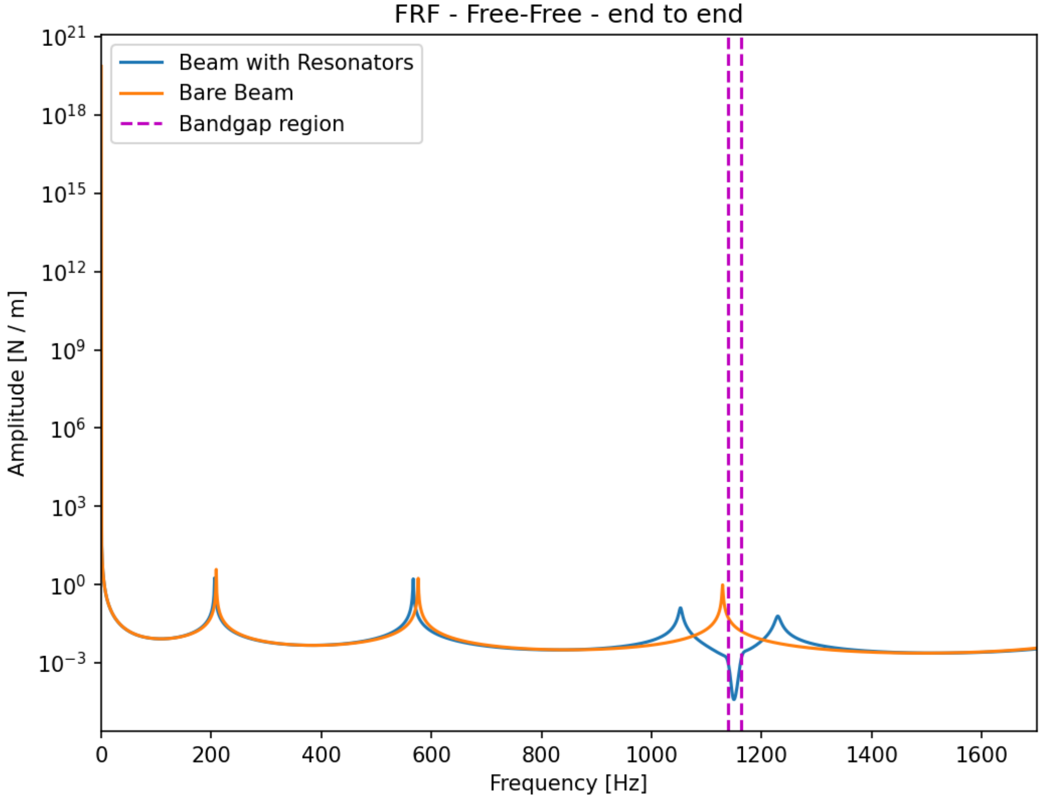
Finite Element Model of beam with ideal resonators

- Beam modelled with 100 Euler-Bernoulli beam elements and 9 ideal SDOF resonators
- Free-free Boundary condition
- Beam has structural damping modelled with a loss coefficient $\eta = 0.001$
- SDOF resonators:
  - Linked to displacement DOF of node correspondent to their location
  - Equally spaced
  - Modelled with a viscous damping ratio $\zeta = 0.005$
- A starting configuration with SDOF parameters is defined:

$$m_{res} = 0.0003968,\ k_{res} = 20573.14,\ f_{res} = 1146 \text{ Hz}$$



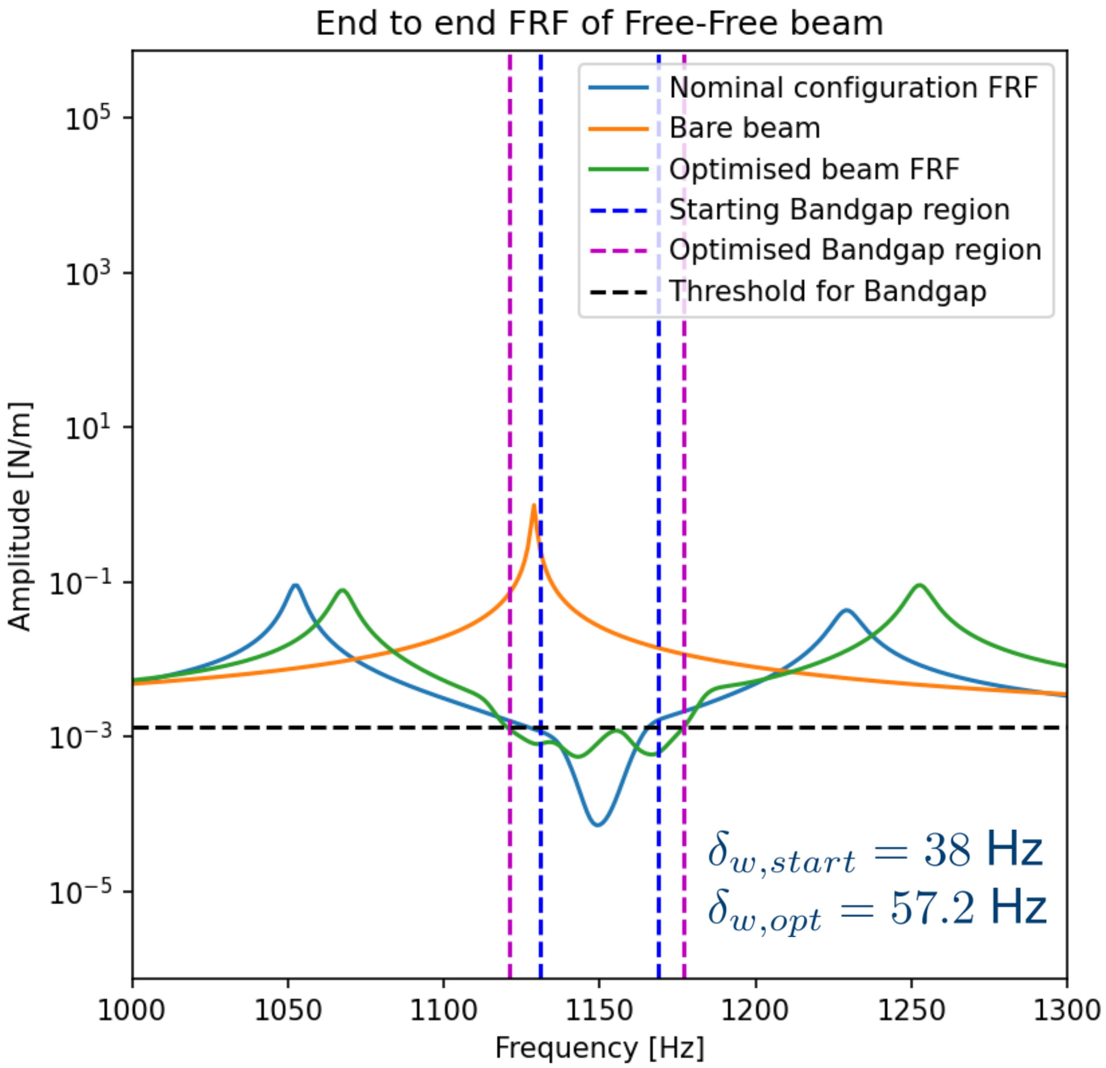
$$F(t) = e^{i\omega t} \qquad \boldsymbol{M}\ddot{q} + \boldsymbol{C}\dot{q} + \boldsymbol{K}q = \boldsymbol{F}(t)$$



Bayesian Optimisation methodology and results

- Bayesian optimisation on an 18-dimensional problem (i.e., k_{res} and m_{res} for each SDOF).
- Interval variability $\pm$ 5% around the starting parameters
- Output is width of bandgap region and the goal is to obtain the maximum width for the given starting configuration.

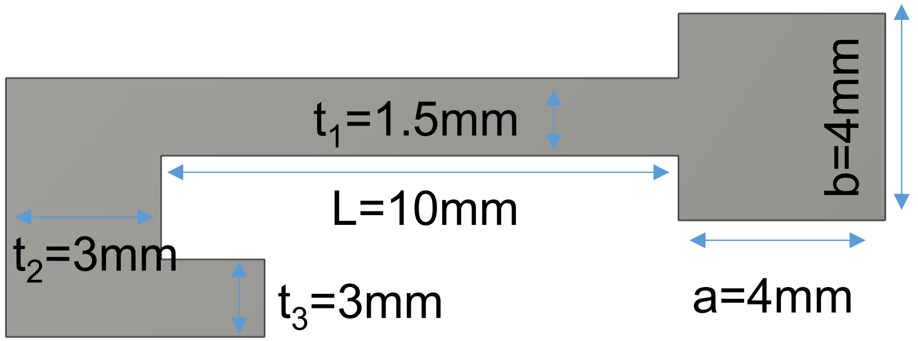
- Implemented with MATLAB's vanilla *bayesopt*, 200 model runs
- Gaussian Process kernel: Matérn 5/2 ARD. Zero mean function.
- Acquisition function to maximise: lower confidence bound.

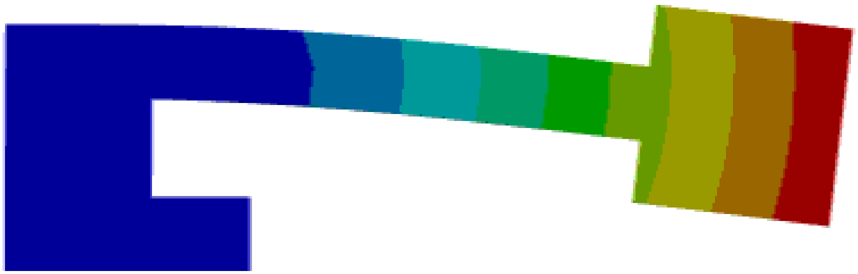


From SDOF to real world resonator

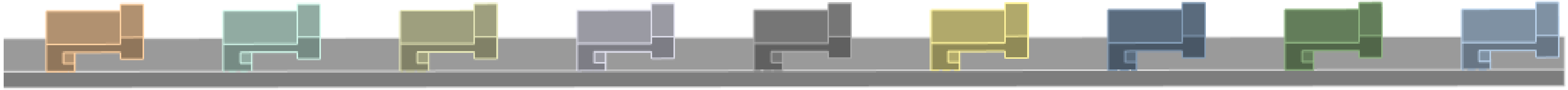
- Resonator design requirements:
    - Only 1 mode shape excited – to emulate SDOF
    - Easy to manufacture with 3D printing
    - Easy to bond to the beam (gluing – araldite)

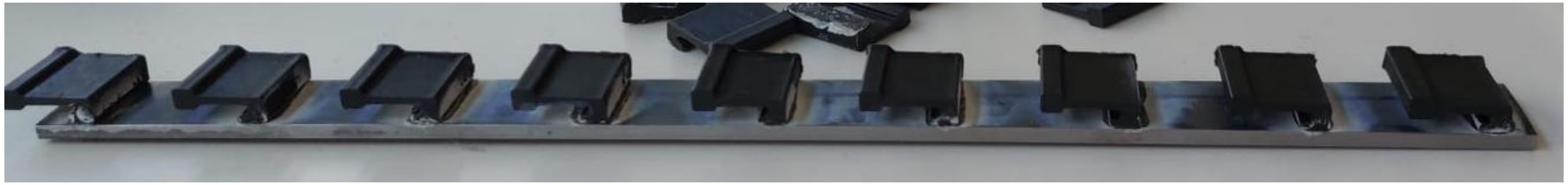
- Ultimaker PLA with material properties: E = 3.019 GPa, ρ = 1240 kg/m^3
- Nominal resonant frequency tuned to 1146 Hz, same as starting configuration used in Optimisation





LRM manufacturing



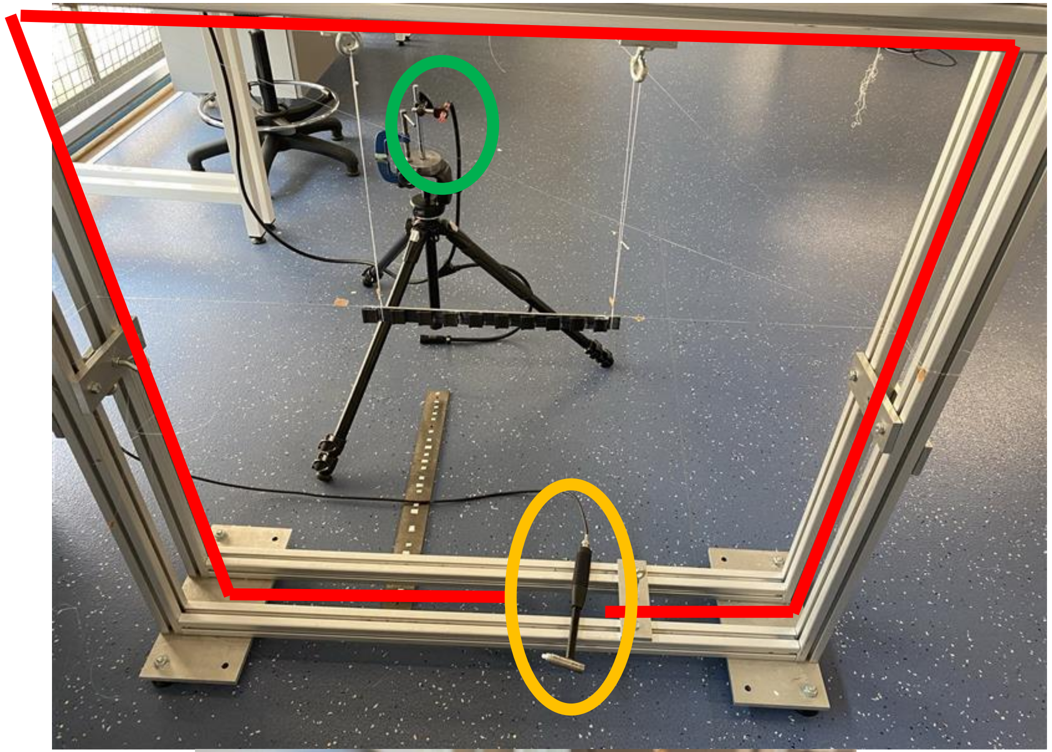


- 2 LRMs built:
  1. Equivalent to the starting configuration before optimisation
  2. Equivalent to the nominal optimised LRM beam (very minimal changes – 10^{-4} m)
     - Each resonator is different (m_{res} and k_{res})

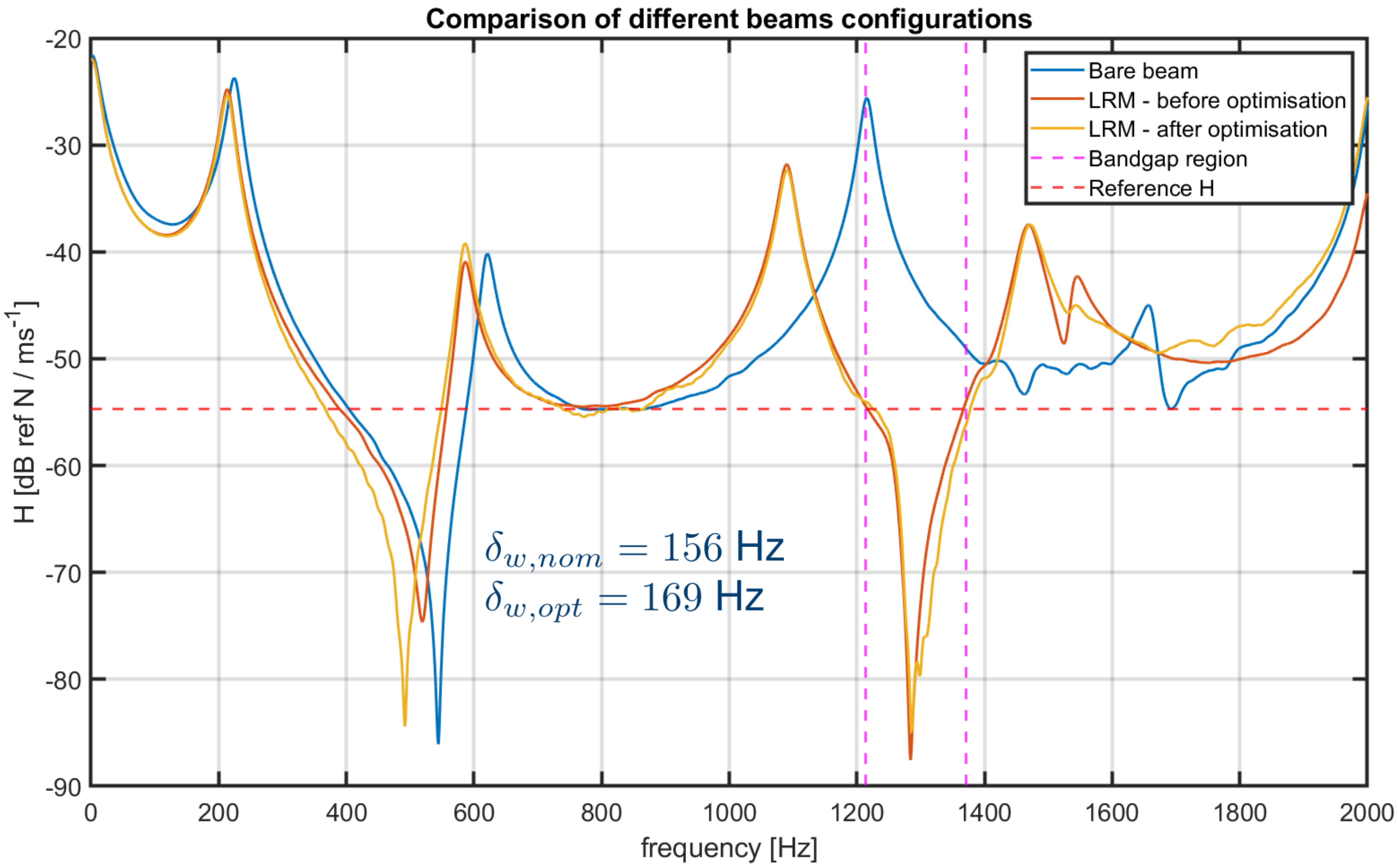
Experimental modal analysis – setup and data processing

- Equipment:
  - Bench for suspension of the beam with strings (Free-Free Boundary condition)
  - Instrumented hammer with IEPE force transducer
  - Laser doppler vibrometer
  - Retroreflective tape

- Data processing:
  - Exponential windowing of force and velocity signals
  - Rectangular windowing of force signals
  - Zero padding of force and velocity signals
  - H1 estimator to average multiple measurements



Experimental results of beam, starting LRM and optimised LRM



Conclusion and further work

- We performed design optimisation on a beam with SDOFs to investigate the influence:
  - Of uncertainties on:
    - Resonator modelling
    - Manufacturing geometrical tolerances and material variability.

- Experimental results show that there is a discrepancy between the modelled and actual LRMs.

Further work:

- Quantifying contribution of each uncertainty in the prototypes
- Develop a novel design optimisation framework that can combine experimental data and experimental uncertainties with physics-based model.

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Thank you!

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Values of m_{res} and k_{res}

m_{res} (kg)	k_{res} (N/m)	f_{res} (Hz)
0.00038	20257.04	1162.116
0.000396	19769.04	1124.416
0.000379	20356.65	1166.515
0.000387	20917.61	1170.622
0.000386	19767.98	1138.46
0.000404	19801.06	1113.605
0.000396	21108.03	1162.198
0.000395	19826.78	1127.198
0.000388	20354.83	1152.822
0.00038	20257.04	1162.116